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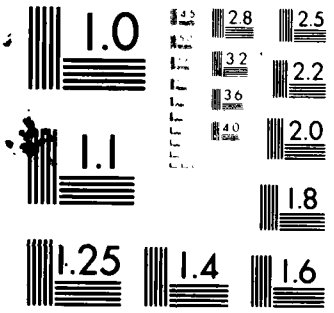

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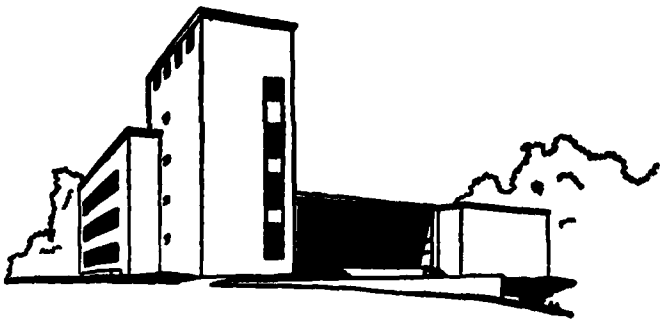

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A NETWORK APPROACH TO COHORT PERSONNEL PLANNING USING CROSS SECTIONAL DATA

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October 1981

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A NETWORK APPROACH TO COHORT PERSONNEL PLANNING USING CROSS SECTIONAL DATA

1. INTRODUCTION

Included in the vast literature of manpower planning models that are solved using linear programming are the goal-programming models in Charnes et al.[1],[2],[4], the maximum effectiveness formulations in Grinold [14] and Thompson [19], and the minimum cost models in Clough et al.[7], Purkiss [18], and Thompson [19]. More recently, network formulations of manpower planning models have gained popularity due to their relatively simple graphic representations and, more importantly, due to the great speed with which their computation can be accomplished. The network approach has been used in the models of Charnes et al.[3],[6] and Thompson [19].

In addition to classifying the mathematical technique used to solve the manpower planning model, (i.e., Linear Programming or Network Programming), we also distinguish between the cross-sectional and longitudinal approaches. The main disadvantage with the standard longitudinal model (Grinold et al.[9],[10],[11],[12]) involves the large amount of historical personnel data that is required for its implementation. Alternatively, the cross-sectional models (Charnes et al.[2],[6], Thompson [19]) require that personnel data be known only at a particular point in time. However, whereas cohort career information is readily available with the longitudinal approach, obtaining such information with a cross-sectional model is difficult.

Using optimal control theory, Gaimon and Thompson [8] circumv[illegible] difficulties by formulating a cohort (longitudinal) manpower planning model that only requ[illegible] cross-sectional data. The model derives the optimal hiring, promotion, separation and retirement policies as functions of time, and a person's organizational age and grade.

In this paper, we reformulate the control theory cohort approach of Gaimon and Thompson as a network model using a technique similar to the one suggested by Price [16], thereby combining the previously cited advantages of network implementation with the added realism achieved by cohort models. In Section 2, the basic notation is defined. With reference to a network diagram, the objective function and the constraints of the basic model are introduced in Section 3. An example of the formulation is presented in Section 4, and is solved using the

network code NETFLO [15]. In Section 5, variations of the basic model are discussed. Finally, the conclusions are given in Section 6.

2. NOTATION

The model uses the following notation:

t : time, $t = 0,1,\ldots,T$, T is the terminal time of planning period.

y : a person's organizational age, $y = 0,1,\ldots,Y$, Y is the age of retirement.

g : grade in the organization, $g = 1,\ldots,G$, G is the highest grade.

$x(t,y,g)$: number of persons of organizational age y, who continue in grade g, at time t. Therefore, $x(t,0,g)$ is the number of persons hired into grade g at time t.

$w(t,g)$: number of persons retiring ($y=Y$) from grade g, at time t, $w(t,g) = x(t,Y,g)$.

$W(t)$: number of persons retiring at time t, $W(t) = \sum_g w(t,g)$.

$z(g)$: number of persons in grade g, at the end of planning period, $t = T$, (end strength), $z(g) = \sum_y x(T,y,g)$.

Z : number of persons in the organization at the end of planning period, $Z = \sum_g z(g)$.

$u(t,y,g)$: number of persons of organizational age y, promoted from grade g into grade $g+1$, at time t.

$v(t,y,g)$: number of persons of organizational age y, separated from grade g, at time t.

$b(t,y,g)$: salary cost of a person of organizational age y, in grade g, at time t (continuation cost).

$r(t,g)$: cost of retiring a person (organizational age Y) from grade g, at time t.

$p(t,y,g)$: cost of promoting a person of organizational age y, from grade g to $g+1$ at time t.

$q(t,y,g)$: cost of separating a person of organizational age y, from grade g, at time t.

$\bar{w}(t,g)$, $\underline{w}(t,g)$: Upper and lower bounds on the goal level interval of $w(t,g)$, respectively.

$\bar{W}(t)$, $\underline{W}(t)$: Upper and lower bounds on the goal level interval of $W(t)$,

respectively.

$\bar{z}(g)$, $\underline{z}(g)$: Upper and lower bounds on the goal level interval of $z(g)$, respectively.

$\bar{Z}$, $\underline{Z}$: Upper and lower bounds on the goal level interval of Z.

P_1 : Penalty per unit deviation in excess of $\bar{w}(t,g)$.

P_2 : Penalty per unit deviation short of $\underline{w}(t,g)$.

P_3 : Penalty per unit deviation in excess of $\bar{W}(t)$.

P_4 : penalty per unit deviation short of $\underline{W}(t)$.

P_5 : Penalty per unit deviation in excess of $\bar{z}(g)$.

P_6 : penalty per unit deviation short of $\underline{z}(t)$.

P_7 : Penalty per unit deviation in excess of $\bar{Z}$

P_8 : penalty per unit deviation short of $\underline{Z}$.

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3. THE MODEL

In the typical goal programming model, a goal level is defined as a particular value. In contrast, we define what we call a goal level interval, which is identified by upper and lower bounds, and which consists of the range of goal level values. Penalties are incurred if an actual level exceeds the respective upper bound of the goal level interval, or if an actual level falls below the respective lower bound of the goal level interval (see Fig.1). We define two penalty functions:

$$G(x - \bar{x}) = [\,|x - \bar{x}| + (x - \bar{x})]/2$$

$$G(\underline{x} - x) = [\,|\underline{x} - x| + (\underline{x} - x)]/2$$

where $\bar{x}$ and $\underline{x}$ are the upper and lower bounds on the variable x. The method by which these penalty functions are incorporated into the network model is explained at the end of this section.

First, stated in words, the objective of this network-goal programming model is to minimize the sum of the penalties incurred due to the deviation between the goal level intervals and the actual levels of

(a) retirements in each grade at each time,

(b) retirements over all grades at each time,

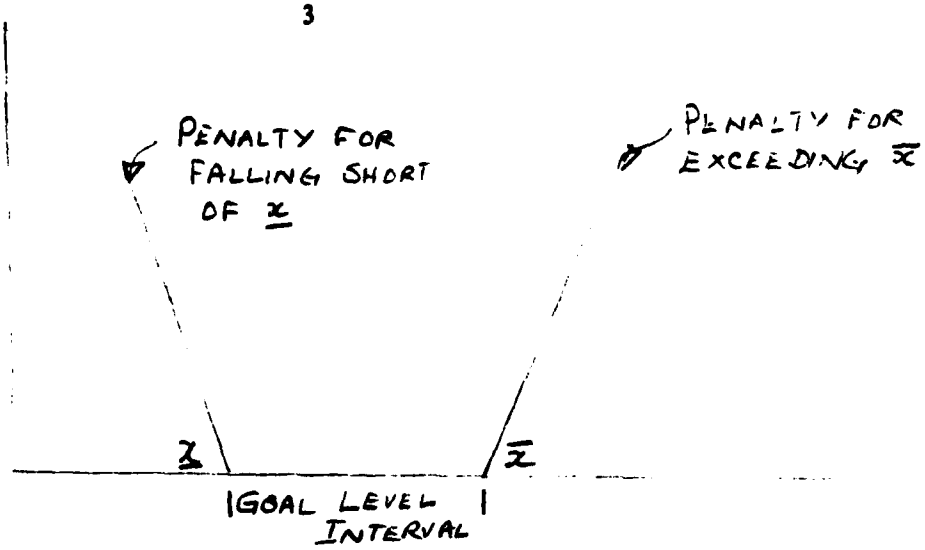

Figure 1: Penalty Function

(c) end strengths in each grade over all organizational ages,

(d) end strength over all grades and organizational ages,

plus the costs incurred during the entire planning period, over all grades and all organizational ages due to

(e) salary(continuations),

(f) promotions,

(g) separations,

(h) the retirement costs incurred during the entire planning period over all the grades.

We distinguish between the costs expressed in (a) and (b) as follows. Whereas (a) represents the sum of the penalty costs due to deviation from the retirement goal level intervals set for each individual grade, (b) depicts the cost of deviating from the goal level interval set for the aggregate level of retirements from all grades. Similarly, whereas (c) represents the sum of the penalty costs due to deviation from the terminal time (end strength) goal level intervals set for each individual grade, (d) depicts the cost of deviating from the goal level interval set for the aggregate level of manpower from all grades at the terminal time. Clearly, if conflicting goals are set for (a) and (b), or for (c) and (d), then the relative per unit penalty costs will determine the dominant goal in the process of minimization. Therefore, the goal level intervals that are used in this model occur at the terminal organizational age and at the terminal time.

Using the notation introduced earlier, and corresponding to the verbal description of the objective given above, we want to minimize

$$\left.\begin{aligned}
&\Sigma_{t,g}\ \{P_1\ (\ G(\ w(t,g) - \bar{w}(t,g)\ )\ ) + \underset{(a)}{P_2}\ (\ G(\ \underline{w}(t,g) - w(t,g)\ )\ )\} + \\
&\Sigma_{t}\ \{P_3\ (\ G(\ W(t) - \hat{W}(t)\ )\ ) + \underset{(b)}{P_4}\ (\ G(\ \underline{W}(t) - W(t)\ )\ )\} + \\
&\Sigma_{g}\ \{P_5\ (\ G(\ z(g) - \bar{z}(g)\ )\ ) + \underset{(c)}{P_6}\ (\ G(\ \underline{z}(g) - z(g)\ )\ )\} + \\
&\{P_7\ (\ G(\ Z - \hat{Z}\ )\ ) + \underset{(d)}{P_8}\ (\ G(\ \underline{Z} - Z\ )\ )\} + \\
&\Sigma_{t,y,g}\ \{\underset{(e)}{b(t,y,g)(x(t,y,g))} + \underset{(f)}{p(t,y,g)(u(t,y,g))} + \underset{(g)}{q(t,y,g)(v(t,y,g))}\} + \\
&\Sigma_{t,g}\ \{\ \underset{(h)}{r(t,g)(w(t,g))}\}.
\end{aligned}\right\}\quad(1)$$

The set of constraints can best be explained with the use of the network diagram in Fig.2. Node 1 refers to the state of the organization at time t-1, for persons of organizational age y-1, and in grade g. Node 2 refers to the state of the organization at time t, for persons of organizational age y, and in grade g. The first of the incoming arcs to node 2,(arc (a)), represents the continuation flow of personnel who remain in grade g from time t-1 (with organizational age y-1) through time t (with organizational age y), x(t-1,y-1,g). The second incoming arc, (arc (b)), indicates the flow of personnel having organizational age y-1 at time t-1 who are promoted from grade g-1 into grade g, during the time interval t-1 to t, u(t-1,y-1,g-1). The three outgoing arcs from node 2 represent, arc(c):the number of persons having organizational age y at time t who continue in grade g from time t to t+1, x(t,y,g); arc(d): the flow of persons having organizational age y at time t, who are separated from grade g during the time interval t to t+1, v(t,y,g); and arc(e): the number of persons in grade g, having organizational age y at time t, who are promoted into grade g+1 during the time interval t to t+1, u(t,y,g). Clearly, the lowest grade can have no incoming flow due to promotions, nor can the highest grade have any outgoing flow due to promotions.

Assuming that the hiring of manpower only occurs into the lowest grade of the organization, we have x(t,0,1) equal to the number of persons hired into grade g at time t, (organizational age equals zero). Due to the short planning periods in which personnel planning models are usually solved, cohorts will not reach retirement during the same planning period that they are hired. Therefore to derive the level of hiring in the current planning period, we only need to consider the goal level intrvals that are set for the end strengths (manpower at the terminal

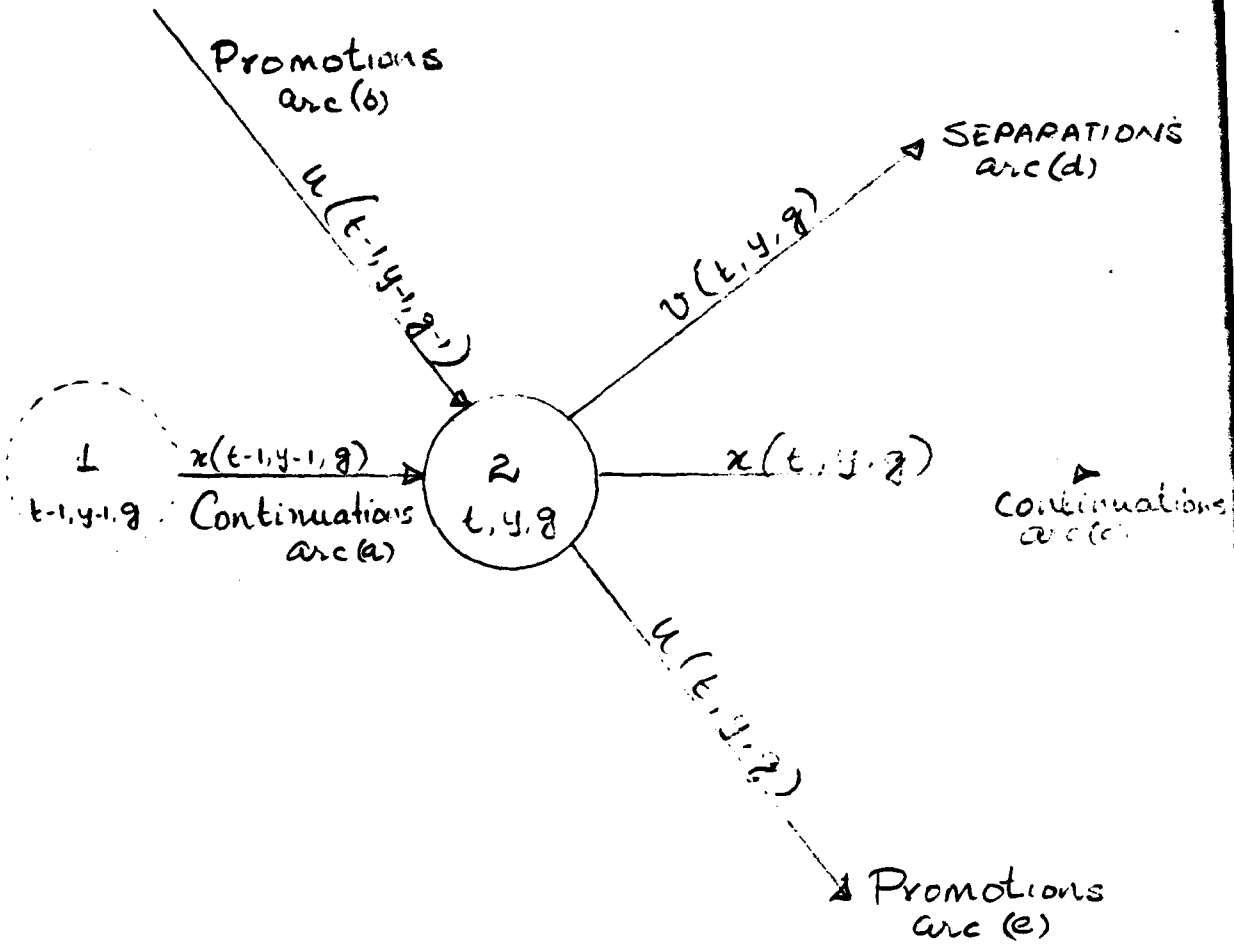

Figure 2: Basic Flows at a Node

time of the planning horizon). Under these conditions, a minimum cost formulation only advocates hiring at the terminal time to meet the end strength goal level intervals in order to reduce the salary (continuation) costs over the entire planning period. To circumvent this difficulty, we define upper bounds on the levels of hiring that can occur at any particular time. Therefore, hiring that is desired at a level in excess of the upper bound at a particular time is forced to occur in the previous periods of the planning horizon. In addition the upper bound on the flow of hiring can reflect the limited supply of labor that is available over time, and the limitations, if any exist, on the hiring expenditure.

The constraints of the model, depicting the permissible flows are written in Equations (2)–(4). The first constraint is the personnel accounting constraint that states that the flow into a

particular node equals the flow out of the node. Therefore, the first constraint equates the sum of the incoming arcs with the sum of the outgoing arcs (see Fig.2). Whereas the accounting constraint examines the flows into and out of a particular node, the equilibrium constraint (3) states that the total flow into the entire network (the number of persons hired and the number of persons already on board), equals the flow out of the network, (the sum of the separations, retirements, and end strengths). The third constraint is the non-negativity constraint.

$$x(t-1,y-1,g) + u(t-1,y-1,g-1) = x(t,y,g) + u(t,y,g) + v(t,y,g)$$

$$\text{for } t = 1,\ldots,T;\ y = 1,\ldots,Y;\ g = 1,\ldots,G \qquad (2)$$

$$\Sigma_t\, x(t,0,1) = \Sigma_{t,y,g}\, v(t,y,g) + \Sigma_{t,g}(\, w(t,g) + \Sigma_g\, z(g) \qquad (3)$$

$$x(t,y,g),\ w(t,g),\ W(t),\ z(g),\ Z,\ u(t,y,g),\ v(t,y,g) \geq 0 \qquad (4)$$

In Fig.3, the method by which goal level intervals and penalty functions are incorporated into the objective function of the basic network model is demonstrated, in a technique similar to the one suggested by Price [16]. Wherever goals exist, a single arc is replaced by three arcs.

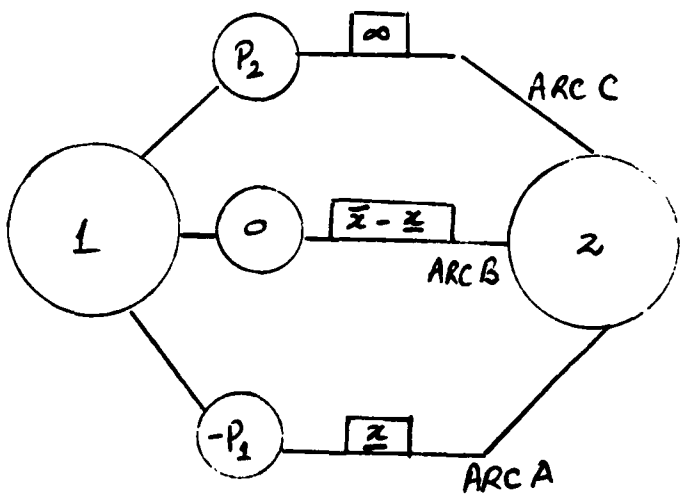

Figure 3: Goals Incorporated in Basic Network

For example, suppose that the goal level interval for x is [$\underline{x}$, $\bar{x}$]. Then the range of flows on each of the three arcs are as follows:

ARC A ==> $0 \leq x \leq \underline{x}$

ARC B ==> $\underline{x} \leq x \leq \bar{x}$

ARC C ==> $\bar{x} \leq x \leq \infty$

Manpower levels in excess of the goal range ($x > \bar{x}$) are penalized on arcs of type ARC C, by a positive amount per unit excess of $\bar{x}$, P_1(see Fig.1). Manpower levels that fall short of the goal level interval ($x < \underline{x}$) are penalized in arcs of type ARC A by a positive amount per unit short of $\underline{x}$, P_2. For example, suppose that we set the goal level interval for the number of retirements from grade g at time t as [50, 75]. In addition, we set the per unit penalty of exceeding the upper bound of the goal level interval at 5, and the per unit penalty of falling short of the goal level intrval lower bound at 8. Then, if the actual number of retirements from grade g at time t is such that $w(t,g) = x(t,Y,g) > 75$, then with $P_1(t,g) = 5$, the total penalty cost is

$$5 (w(t,g) - 75) = 5 \text{ (flow on ARC C)}$$

If the actual number of retirements from grade g at time t is such that $w(t,g) < 50$ then the total penalty cost is

$$8 (50 - w(t,g)) = 450 - 8 \text{ (flow on ARC A)}$$

Therefore, we set $P_2(t,g) = -8$, and we note that if the flow on ARC A is not at its upper bound in the final solution, then we add 450 to the value of the optimal solution. Arcs of type ARC B represent the goal level intervals and have zero costs assigned to them, except when organizational age equals Y, in which case, the costs on the arcs reflect the retirement costs.

Since our model is a minimum cost formulation, the negative per unit penalty costs associated with arcs of type ARC A have the effect of setting the flows on these arcs at their upper bounds whenever possible in the final solution. If the flow on an arc of type ARC A is not at its upper bound in the final solution, then we add a correction term equal to the product of the per unit penalty and the lower bound on the goal level interval to the value of the optimal solution.

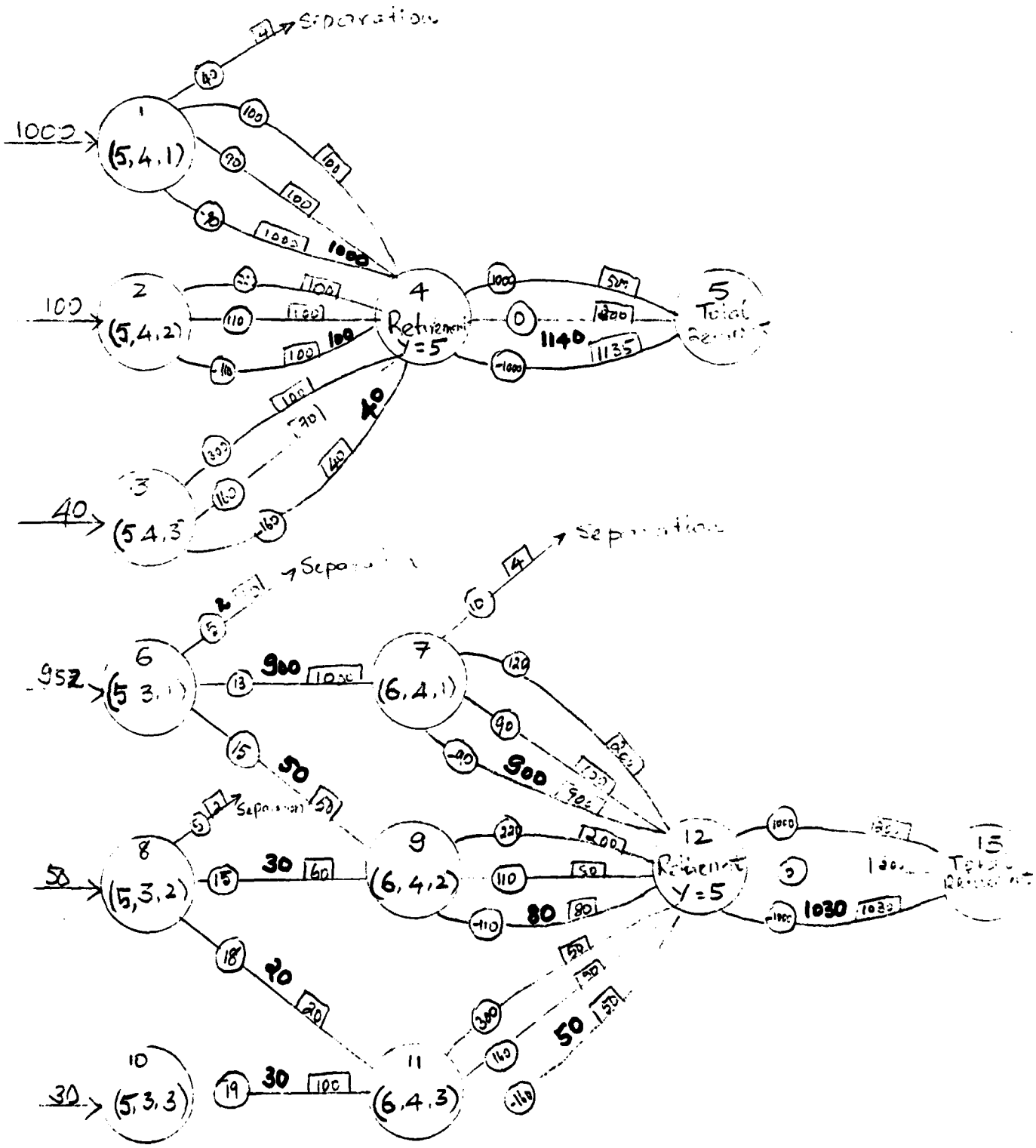

Figure 4A: The Retirement Problem

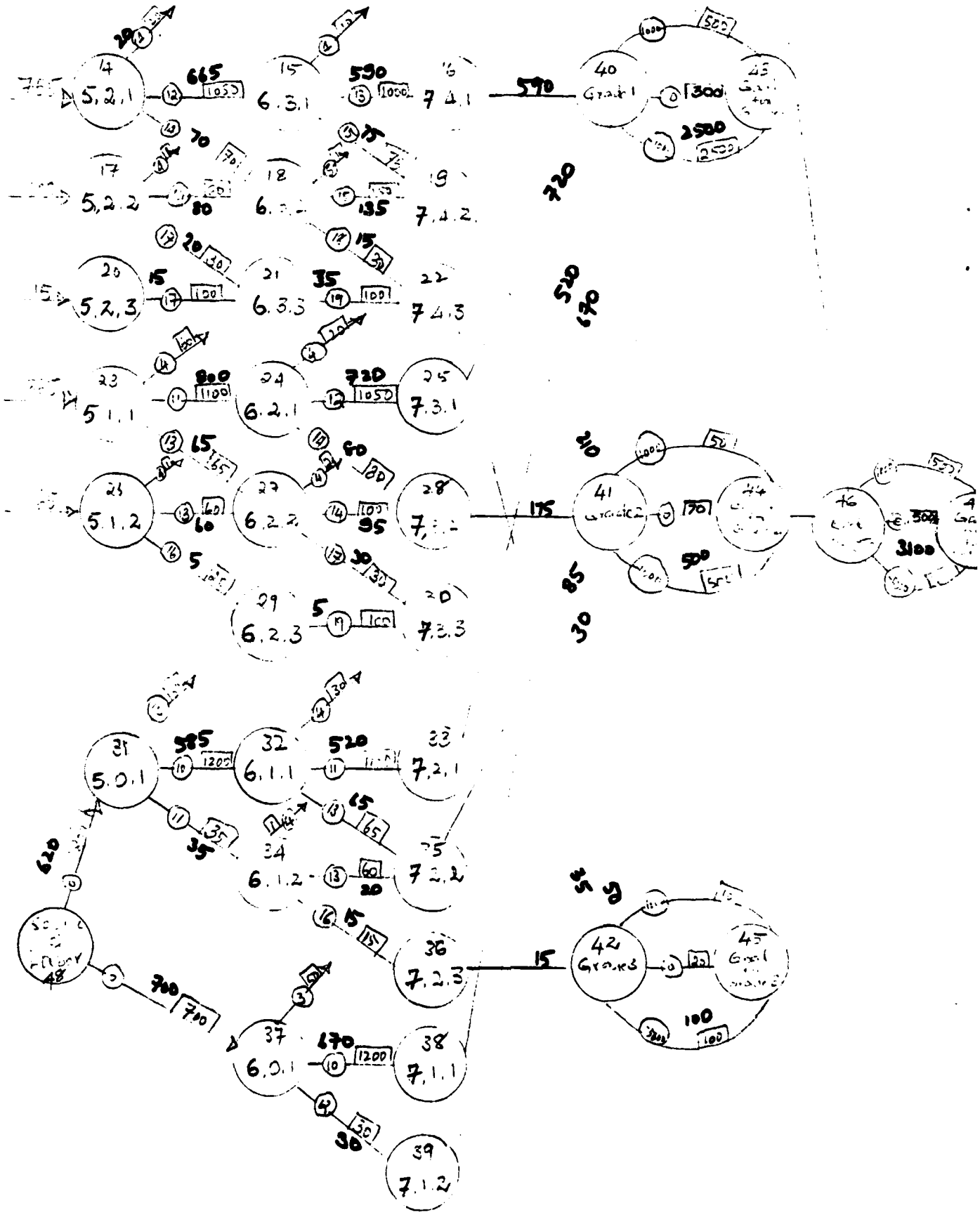

Figure 46: The Hiring Problem

4. EXAMPLE

In this example, we consider an organization having three grades and a five year retirement age. Therefore, at the current time, t = 5, we have four cohort groups on board, (y = 1,2,3,4). The number of persons in each of these cohort groups is:

```
x(5,1,1) = 865
x(5,1,2) = 65

x(5,2,1) = 755
x(5,2,2) = 100
x(5,2,3) = 15

x(5,3,1) = 952
x(5,3,2) = 50
x(5,3,3) = 30

x(5,4,1) = 1000
x(5,4,2) = 100
x(5,4,3) = 40
```

Assuming a planning period of two years, we derive the optimal hiring, continuation, promotion, separation, and retirement levels for the current period, t = 5, and for the next period, t = 6. The upper bounds on hiring in periods 5 and 6 are 800 and 700, respectively. The retirement goal level intervals for cohorts of age four at time five are (1000,1100), (100,200), and (40,110) for grades 1, 2, and 3, respectively. For cohorts of age three at time five, the retirement goal level intervals are (900,1000), (80,160), and (50,100) for grades 1, 2, and 3, respectively. Clearly, retirement goal level intervals need only be defined for these two cohort groups in our two period planning horizon. The end strength goal level intervals for persons in grades 1, 2, and 3 are (2500,2800), (500,570), and (100,120), respectively, and the total end strength goal level interval for persons in all grades is (3000,3500).

Fig.4A and Fig.4B illustrate the optimal solution of this example. Due to the high penalty assigned to any deviation from the retirement goal level intervals, we see that all retirement goals are met in the final solution. Similarly, all end strength goals are met as well. However, while hiring occurs at a level equal to its upper bound in period 6, (x(6,0,1) = $\bar{x}$ (6,0,1) = 700), the level of hiring in period 5 falls short of the respective upper bound, (x(5,0,1) = 620 < $\bar{x}$ (5,0,1) = 800).

In the table we compare the results obtained when goal levels are decreased and increased by 10% and also when the available labor supply is reduced by 50%. When goal levels are decreased by 10%, the optimal solution advocates hiring less number of persons in the fifth period, and all the goal levels for the individual grades are met at their lower bounds while the total end strength is attained inside the goal level range. When goal levels are increased by

10%, the total end strength is achieved but the individual grade goals fall short. This is especially true for Grade 3, since its goal level cannot be attained by hiring more persons at current period. When there is a shortage in labor supply, all the available persons are hired, but the Grade 1 goal and the total end strength goal fall short.

	GOAL LEVEL	LEVEL ATTAINED	LEVEL ATTAINED WITH 10% DECREASE IN GOAL LEVELS	LEVEL ATTAINED WITH 10% INCREASE IN GOAL LEVELS	LEVEL WITH 50% LABOR SUPPLY
GRADE 1	2500-2800	2500	2250	2700	2200
GRADE 2	500-570	500	450	535	500
GRADE 3	100-120	100	90	65	100
TOTAL	3000-3500	3100	2790	3300	2800
NUMBER HIRED IN YEAR 5		620	312	800	500
NUMBER HIRED IN YEAR 6		700	700	700	500

5. VARIATIONS OF THE BASIC MODEL

Depending on the objectives of the management, many variations of the basic model exist. Below, we briefly describe some of these extensions.

(a). In order to consider the problem of maximizing organizational effectiveness, assuming that effectiveness is both linear and quantifiable, we simply assign negative values to the penalty functions to reflect effectiveness rather than cost.

(b). Goal level intervals for continuation, promotion, and separation can be added to the model by substituting multiple arcs for single arcs as is illustrated in Fig.3.

(c). Piecewise linear penalty functions can easily be accommodated by introducing multiple arcs (see Fig.3) between the appropriate nodes in the network.

(d).The organization might be interested in finding out what would happen if persons about to retire are offered a bonus to stay on, and the effect of such a possibility on the goal

levels. This would be the case when there is scarcity of possible recruits.

6. CONCLUSIONS

In this paper we have presented a network cohort personnel planning model that is solved using cross-sectional data. In essence, this formulation captures the realism achieved by the longitudinal approach, the ease in gathering data offered by the cross-sectional models, and the speed and simplicity of computation available with the network solution method.

We have introduced goal level intervals which are identified by upper and lower bounds and which consist of ranges of goal level values. Penalties are incurred if actual levels exceed the upper bounds or fall short of the lower bounds. We introduce a new technique with which we are able to incorporate the goal level intervals into the objective function. We set goal level intervals for the levels of retirement of cohorts during the entire planning period, and for the levels of all cohorts in the organization at the terminal time of the planning period, (end strengths).

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